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Self-tuning solution of the cosmological constant problem with antisymmetric tensor field

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Abstract

We present a self-tuning solution of the cosmological constant problem with one extra dimension which is curved with a warp factor. To separate out the extra dimension and to have a self-tuning solution, a three index antisymmetric tensor field is introduced with the $1/H^2$ term in the Lagrangian. The standard model fields are located at the $y = 0$ brane. The existence [1] of the self-tuning solution (which results without any fine tuning among parameters in the Lagrangian) is crucial to obtain a vanishing cosmological constant in a 4D effective theory. The de Sitter and anti de Sitter space solutions are possible. The de Sitter space solutions have horizons. Restricting to the spaces which contain the $y = 0$ brane, the vanishing cosmological constant is chosen in the most probable universe. For this interpretaion to be valid, the existence of the self-tuning solution is crucial in view of the phase transitions. In this paper, we show explicitly a solution in case the brane tension shifts from one to another value. We also discuss the case with the H^2 term which leads to one-fine-tuning solutions at most.

I. INTRODUCTION

The cosmological constant problem [2] is probably the most important clue to the physics at the Planck scale. Most attempts toward solutions of the cosmological constant problem introduce additional ingredients [3, 4, 5, 6, 7, 8], except the wormhole and anthropic solutions of the problem [6, 9].

The wormhole solution is based on the probabilistic interpretation that the probability to have a universe with a vanishing cosmological constant is the largest [6]. But in the evolving universe where the true vacuum is chosen as the universe cools down, the probabilistic interpretation in the early universe is in question since at a later epoch additional constant may be generated by spontaneous symmetry breaking. For this interpretation to make sense, there must exists a *self-tuning solution*. [The *self-tuning* solution is defined as the flat space solution without any fine-tuning of parameters in the action.] The anthropic interpretation [9] for the small cosmological constant is a working proposal, but does not answer the fundamental question, "Can we explain the vanishingly small constant from the fundamental parameters in the theory?"

On the other hand, Hawking's probably vanishing cosmological constant [5] relies on an undetermined integration constant. He showed that the wave function for a flat universe is infinitely large compared to non-flat universes. Thus, if there exists an undetermined integration constant, the phase transition will end probably to a flat universe. However, his original proposal with a three index antisymmetric tensor field does not introduce any dynamics in the 4D space-time and the integration constant is just another cosmological constant [5, 8]. Witten [3] also argued that the existence of an undetermined integration constant may be a clue to the understanding of the flat universe, since a many-body theory may provide an explanation from a one constant universe to another constant universe [10]. Thus, in the absence of a self-tuning solution of the cosmological constant, we regard that the cosmological constant problem in Hawking's scenario still remains as an unsolved problem.

Therefore, it is worthwhile to search for any new solution of the cosmological constant problem. Since the gravitational law beyond the Planck scale is not a settled issue at present, we can look for solutions even in models with drastically different gravity beyond the Planck scale. If the cosmological constant problem is solved in models with a different gravity, then the new interaction can be studied extensively whether it leads to another inconsistency in theory or in phenomenology. In this spirit, there have been attempts to understand the cosmological constant in the 5 dimensional (5D) world with the fifth dimension y compactified [11] (RSI model) or uncompactified [12] (RSII model). It seems that there can be a way to understand the cosmological constant problem [13] in these RS type models since, RSI model for example, the nonvanishing brane tensions k_1 and k_2 and bulk cosmological constant k can lead to a flat space solution if these parameters satisfy two fine-tuning conditions $k_1 = k = -k_2$. Therefore, the first step to understand the cosmological constant is to find the flat space solutions without any fine-tuning between parameters in the action.

With the fifth dimension compactified (RSI), the attempts to solve the cosmological constant has failed so far. The original try to find a flat space solution without any fine-tuning [14] had a singularity, and taking the singularity into account by putting a brane there reproduced a fine-tuning condition [15] even though the two conditions have been reduced to just one. Therefore, the first step toward a solution of the cosmological constant in the RS type models is to have flat space solutions without any fine-tuning between parameters in the Lagrangian. Restricting just to an exponentially small cosmological constant, it has

been pointed out that it is possible with two or more branes [16, 17, 18].

Recently, the needed flat space solutions in the RSII type background have been found [1]. With a smeared-out brane a similar attempt led to a self-tuning solution [19]. In the RS type models, the brane(s) has a special meaning in the sense that the matter fields can reside at the brane only. However, the effective gravitational interaction of the matter fields is obtained after integrating out the fifth dimension y. For this effective theory to make sense, we must require that:

(i) the metric is well-behaved in the whole region of the bulk, and

(ii) the resulting 4D effective Planck mass is finite.

The condition (i) is to find a solution without a singularity in the region defined. In this regard, if the warp factor vanishes at say $y = y_m$, then y_m becomes the horizon and the universe connected to the matter brane is up to $y = y_m$. In the RSI models, it is a disaster if $0 < y_m < \frac{1}{2}$ is between the two branes located at $y = 0$ and $y = 1/2$. This is because one needs both branes for the consistency in the RSI models. But in the RSII models, if there exists y_m then one can consider the space only for $-y_m < y < y_m$ if y_m is not a naked singularity. Indeed, it can be shown that it is consistent to consider the space up to this point only by calculating the effective cosmological constant by integrating y to y_m. The localized gravity condition (ii) restricts the solutions severely, since the y integration in some solutions would give a divergent quantity for M_P or M_P get more important contributions as $y \to \infty$.

The Einstein-Hilbert action with a bulk cosmological constant and a brane tension in the RSII model does not allow a self-tuning solution. Addition of the Gauss-Bonnet term [20] does not improve the situation, and also it does not help to regularize a naked singularity in the self-tuning model with a bulk scalar [14]. But addition of the three index antisymmetric tensor field A_{MNP} (and the field strenth H_{MNPQ} and $H^2 \equiv H_{MNPQ}H^{MNPQ}$) allows a self-tuning solution [1].

In this paper, we discuss the RSII model with antisymmetric tensor field added. The case with $1/H^2$ term allowing the self-tuning solution is the main motivation for this extensive study. H_{MNPQ} has been considered before in connection with the cosmological constant problem [3] and the possible compactification of the seven internal space in the 11D supergravity [21]. Even in the 5D Randall-Sundrum model H_{MNPQ} is useful to separate the 4 dimensional space. The $1/H^2$ term looks strange, but the consideration of the energy-momentum tensor would require a nonvanishing $\langle H_{MNPQ} \rangle$, triggering the separation of the extra dimension from the 4 dimensional space. Below the Planck scale, we consider that the action is an effective theory. Above the Planck scale, we consider that the quantum gravity effects may be very important, but at present the final form for quantum gravity is not known yet.

In this paper, *the self-tuning solution means that it does not need a fine-tuning between the parameters in the action*, which is a progress toward understanding the cosmological constant problem. The self-tuning solutions are found from time independent Einstein equations. However, the existence of the self-tuning solution alone does not solve the cosmological constant problem completely. It is because if the ansatz for a *time dependent metric* allows, for example, the de Sitter space solutions with the antisymmetric tensor field added then choosing the flat space is simply choosing a boundary condition. However, the existence of the *self-tuning solution* and the probabilistic interpretation for the wave function of the universe can provide a logical understanding of the vanishing cosmological constant even in this case [5]. Suppose that we start from a flat universe from the beginning *á la* the

wormhole interpretation of the vanishing cosmological constant. But this interpretation alone may encounter a difficulty when the phase transitions such as the electroweak phase transition or the QCD phase transition add a nonvanishing cosmological constant at a later epoch. However, the existence of the self-tuning solution chooses the flat space solution out of numerous possibilities when the universe goes through these phase transitions. If there exists a self-tuning solution, then the wormhole interpretation chooses the vanishing cosmological constant even after these phase transitions.

We find that the $1/H^2$ term *does* always allow de Sitter space solutions with localized gravity. In the RS model, however, a *negative* brane tension ($\Lambda_1 < 0$) does not allow a localized gravity in the de Sitter space [22], but allows only a nonlocalized gravity. There are arguments excluding these nonlocalized gravity [23], and in the RSII model for a negative brane tension one may exclude the de Sitter space solutions. However, the nonlocalized gravity cannot become a strong argument for the vanishing cosmological constant. It simply means that the de Sitter space solution with nonlocalized gravity cannot materialize to our universe. At some point, we may invoke an anthropic principle or turn to a probabilistic interpretation. Namely, as long as there exist solutions for nonzero cosmological constants whether there results a localized gravity or not, we need a probabilistic interpretaion. For a probabilistic interpretation, the existence of the self-tuning solution is crucial in choosing the flat universe.

In Sec. II, we present the flat space solutions with $1/H^2$ term and with H^2 term. It is shown that $1/H^2$ term allows the flat space self-tuning solution but H^2 term allows at most one-fine-tuning solutions. De Sitter space and anti de Sitter space solutions are also commented. In Sec. III, it is shown that the $1/H^2$ term allows the anti de Sitter and de Sitter space solutions. We discuss the horizons appearing in our solutions. We also discuss how the universe chooses the vanishing cosmological constant. In Sec. IV, we present a time-dependent solution such that the 4D space time remains *flat* when the brane tension shift instantaneously to another value. Sec. V is a conclusion.

II. THE STATIC SOLUTIONS

The five dimensional space is composed of the bulk and a 3-brane located at $y = 0$ where y is the fifth coordinate. We assume that matter fields live in the brane. For studying the gravity sector, we include the three index antisymmetric tensor field A_{MNP} whose field strength is denoted as H_{MNPQ}, where $M, N, \cdots = 0, 1, 2, 3, 5(\equiv y)$. We find that there exist solutions for different bulk cosmological constants at $y < 0$ and $y > 0$. But for simplicity of the discussion, we will introduce a Z_2 symmetry so that the bulk cosmological constant is universal. In this section, we summarize the self-tuning solution [1] with the time independent metric. But for comparison we briefly comment the time dependent metric, i.e. the de Sitter space and anti de Sitter space solutions with the $1/H^2$ term. We also present one-fine-tuning solutions for H^2 term with time independent metric, and compare with the other known tuning solutions [14, 19].

A. A self-tuning solution of the cosmological constant with $1/H^2$

A self-tuning solution exists for the following action,

$$S = \int d^4x \int dy \sqrt{-g}\left(\frac{1}{2}R + \frac{2\cdot 4!}{H_{MNPQ}H^{MNPQ}} - \Lambda_b + \mathcal{L}_m\delta(y)\right) \tag{1}$$

which will be called the KKL model [1]. Here we set the fundamental mass parameter M as 1 and we will recover the mass M wherever it is explicitly needed. We assume a Z_2 symmetry of the warp factor solution, $\beta(-y) = \beta(y)$. The sign of the $1/H^2$ term is chosen such that at the vacuum the propagating field A_{MNP} has a standard kinetic energy term.

The action contains the $1/H^2$ term which does not make sense if H^2 does not develop a vacuum expectation value. Since the cosmological constant problem is at the bottom of most cosmological application of particle dynamics, it is worthwhile to study any solution to the cosmological constant problem. We note that this problem has led to so many interesting but unfamiliar ideas [3, 4, 5, 7]. Therefore, any new idea in the possible interpretation of the cosmological constant problem is acceptable at this stage. In fact, we found a very nice solution with the above action and hence we propose the action (1) as the fundamental one in gravity. Being a part of gravity, we do not worry about the renormalizability at this stage.

Flat space solution The ansatz for the metric is taken as

$$ds^2 = \beta^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{2}$$

where $(\eta_{\mu\nu}) = \text{diag.}(-1, +1, +1, +1)$. Then Einstein tensors are,

$$\begin{aligned} G_{\mu\nu} &= g_{\mu\nu}\left[3\left(\frac{\beta'}{\beta}\right)^2 + 3\left(\frac{\beta''}{\beta}\right)\right], \\ G_{55} &= 6\left(\frac{\beta'}{\beta}\right)^2. \end{aligned} \tag{3}$$

where prime denotes differentiation with respect to y. With the brane tension Λ_1 at the $y = 0$ brane and the bulk cosmological constant Λ_b, the energy momentum tensors are

$$T_{MN} = -g_{MN}\Lambda_b - g_{\mu\nu}\delta^\mu_M\delta^\nu_N\Lambda_1\delta(y) + 4\cdot 4!\left(\frac{4}{H^4}H_{MPQR}H_N{}^{PQR} + \frac{1}{2}g_{MN}\frac{1}{H^2}\right). \tag{4}$$

The specific form for $H^2 \equiv H_{MNPQ}H^{MNPQ}$ in Eq. (1) makes sense only if H^2 develops a vacuum expectation value at the order of the fundamental mass scale. Because of the gauge invariant four index H_{MNPQ}, four space-time is singled out from the five dimensions [21]. The four form field is denoted as $H_{\mu\nu\rho\sigma}$,

$$H_{\mu\nu\rho\sigma} = \sqrt{-g}\frac{\epsilon_{\mu\nu\rho\sigma}}{n(y)} \tag{5}$$

where $\mu, \cdots$ run over the Minkowski indices 0, 1, 2, and 3. With the above ansatz, the field equation for the four form field is satisfied,

$$\partial_M\left(\sqrt{-g}\frac{H^{MNPQ}}{H^4}\right) = 0. \tag{6}$$

There exists a solution for $\Lambda_b < 0$. The two relevant Einstein equations are the (55) and ($\mu\mu$) components,

$$6\left(\frac{\beta'}{\beta}\right)^2 = -\Lambda_b - \frac{\beta^8}{A} \tag{7}$$

$$3\left(\frac{\beta'}{\beta}\right)^2 + 3\left(\frac{\beta''}{\beta}\right) = -\Lambda_b - \Lambda_1\delta(y) - 3\frac{\beta^8}{A} \tag{8}$$

where A is a positive constant in view of Eq. (5). It is easy to check that Eq. (8) in the bulk is obtained from Eq. (7) for any Λ_b, Λ_1, and A. This property is of the specialty of the H_{MNPQ} field. Near B1(the $y = 0$ brane), the δ function must be generated by the second drivative of β. The Z_2 symmetry, $\beta(-y) = \beta(y)$, implies $(d/dy)\beta(y)|_{0^+} = -(d/dy)\beta(y)|_{0^-}$. Thus,

$$\frac{d^2}{dy^2}\beta(|y|) = \frac{d^2}{dy^2}\beta(|y|)\Big|_{y\neq 0} + 2\delta(y)\frac{d}{d|y|}\beta(|y|). \tag{9}$$

This δ-function condition at B1 leads to a boundary condition

$$\frac{\beta'}{\beta}\Big|_{y=0^+} \equiv -k_1, \tag{10}$$

where we define k's in terms of the bulk cosmological constant and the brane tension,

$$k \equiv \sqrt{-\frac{\Lambda_b}{6}}, \quad k_1 \equiv \frac{\Lambda_1}{6}. \tag{11}$$

Let us find a solution for the bulk equation Eq. (7) with the boundary condition Eq. (10). We define a in terms of A,

$$a = \sqrt{\frac{1}{6A}}. \tag{12}$$

The solution of Eq. (7) consistent with the Z_2 symmetry is

$$\beta(|y|) = \left(\frac{k}{a}\right)^{1/4}[\cosh(4k|y| + c)]^{-1/4}, \tag{13}$$

where c is an integration constant to be determined by the boundary condition Eq. (10). This solution, consistent with Condition (i), is possible for any value of the brane tension Λ_1. Note that c can take any sign. This solution gives a localized gravity consistent with the above Condition (ii). The boundary condition (10) determines c in terms of Λ_b and Λ_1,

$$c = \tanh^{-1}\left(\frac{k_1}{k}\right) = \tanh^{-1}\left(\frac{\Lambda_1}{\sqrt{-6\Lambda_b}}\right). \tag{14}$$

A schematic shape of $\beta(y)$ is shown in Fig. 1.

The effective 4D Planck mass is finite

$$M_{P,\text{eff}}^2 = 2M^3\left(\frac{k}{a}\right)^{1/2}\int_0^\infty dy\frac{1}{\sqrt{\cosh(4ky+c)}} = \frac{M^3}{\sqrt{2ka}}F\left[\alpha, \frac{1}{\sqrt{2}}\right]_0^\infty$$

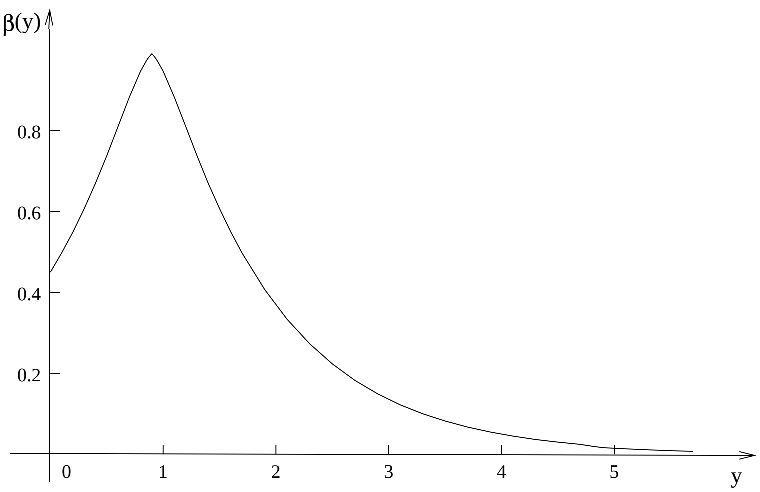

FIG. 1: $\beta(y)$ as a function of y for the flat space ansatz. It is plotted for $k=1$ and $a=1$.

$$= \frac{M^3}{\sqrt{2ka}} \int_{\sqrt{1-(\cosh(c))^{-1}}}^{1} \frac{dx}{\sqrt{(1-x^2)(1-\frac{1}{2}x^2)}}. \tag{15}$$

Here $F(\alpha, r)$ is the elliptic integral of the first kind and

$$\alpha = \sin^{-1}\sqrt{(\cosh(4ky+c)-1)/(\cosh(4ky+c))}. \tag{16}$$

Note that the Planck mass is given in terms of the integration constant a, or the integration constant is expressed in terms of the fundamental mass M and the 4D Planck mass $M_{P,\mathrm{eff}}$,

$$a = \left(\frac{M^3}{M_{P,\mathrm{eff}}^2 \sqrt{2k}} F\left[\alpha, \frac{1}{\sqrt{2}}\right]_0^\infty \right)^2. \tag{17}$$

Curved space solution   The curved space solution is a time-dependent solution which will be discussed in the KKL model of Sec. III.

Localization of gravity and no tachyon   The perturbed metric is

$$ds^2 = (\beta^2 \eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu + dy^2 \tag{18}$$

where we chose the Gaussian normal condition, $h_{5\mu} = h_{55} = 0$. With the transverse traceless gauge, $\partial^\mu h_{\mu\nu} = h^\mu_\mu = 0$, and by a separation of variables such as $h_{\mu\nu} = \epsilon_{\mu\nu} e^{ipx} \psi(y)$ ($p^2 = -m^2$), we obtain the following linearized equations without matter on the brane,

$$\left[-\frac{1}{2}\beta^{-2}m^2 - \frac{1}{2}\partial_y^2 + 2k^2 - 2k_1\delta(y) - 6a^2\beta^8 \right]\psi(y) = 0 \tag{19}$$

where

$$k_1 \equiv \frac{\Lambda_1}{6}, \tag{20}$$

$$a \equiv \frac{1}{\sqrt{6A}}. \tag{21}$$

Here we make a change of variable by $z=\int^y \frac{dy}{\beta(y)} \equiv \int^y du\, e^{-C(u)}$ and $\psi(y)=\beta^{1/2}\hat{\psi}(z)$ and then obtain the Schrödinger-like equation as follows,

$$\Big[-\frac{\partial^2}{\partial z^2}+V(z)\Big]\hat{\psi}(z)=m^2\hat{\psi}(z) \tag{22}$$

where

$$V(z) = \frac{15}{4}k^2\beta^2-3k_1\beta\delta(z)-\frac{39}{4}a^2\beta^{10} \tag{23}$$

$$= \frac{9}{4}\Big(\frac{\partial C}{\partial z}\Big)^2+\frac{3}{2}\Big(\frac{\partial^2 C}{\partial z^2}\Big). \tag{24}$$

Therefore, we can see that $m^2\geq 0$, i.e., there is no tachyon state near the background, by regarding the above equation as a supersymmetric quantum mechanics [19],

$$Q^\dagger Q\hat{\psi}(z)\equiv\Big(\partial_z+\frac{3}{2}\frac{\partial C}{\partial z}\Big)\Big(-\partial_z+\frac{3}{2}\frac{\partial C}{\partial z}\Big)\hat{\psi}(z)=m^2\hat{\psi}(z). \tag{25}$$

Moreover, there appears the localization of gravity on the brane as expected from the finite 4D Planck mass, due to one bound state of massless graviton, $\hat{\psi}_0(z)\sim e^{3C/2}=\beta^{3/2}$, that is, $\psi_0(y)\sim\beta^2$. Note that the zero mode solution $\psi_0(y)$ automatically satisfies the boundary condition $(\partial_y+2k_1)\psi|_{y=0+}=0$ from Eq. (19).

B. One-fine-tuning solutions of the cosmological constant with H^2 term

In this section, we obtain just the flat space solution for the H^2 term in the action,

$$S=\int d^4x\int dy\sqrt{-g}\left(\frac{M^3}{2}R-\frac{M}{2\cdot 4!}H_{MNPQ}H^{MNPQ}-\Lambda_b+\sum_i\mathcal{L}_m^{(i)}\delta(y-y_i)\right). \tag{26}$$

The ansatze for the metric and the four form field are taken also as given in Eq. (2) and Eq. (5), respectively. Thus the field equation for A_{MNP} is trivially satisfied again,

$$\partial_M\Big[\sqrt{-g}H^{MNPQ}\Big]=0. \tag{27}$$

The two relevant Einstein equations are the (55) and ($\mu\mu$) components,

$$6\left(\frac{\beta'}{\beta}\right)^2 = -\Lambda_b+\frac{A}{\beta^8} \tag{28}$$

$$3\left(\frac{\beta'}{\beta}\right)^2+3\left(\frac{\beta''}{\beta}\right) = -\Lambda_b-\Lambda_1\delta(y)-\Lambda_2\delta(y-y_c)-\frac{A}{\beta^8} \tag{29}$$

where $A/\beta^8\equiv 1/2n^2$ expressed in terms of a 'positive' constant A.

The solutions of Eq. (28) and (29) are

$$\text{for}\quad \Lambda_b<0 \;:\; \beta(|y|) = \left(\frac{a}{k}\right)^{1/4}[\sinh(|4k|y|+c|)]^{1/4} \tag{30}$$

$$\text{for}\quad \Lambda_b>0 \;:\; \beta(|y|) = \left(\frac{a}{k}\right)^{1/4}[\sin(|4k|y|+c|)]^{1/4} \tag{31}$$

$$\text{for}\quad \Lambda_b=0 \;:\; \beta(|y|) = (|4a|y|+c|)^{1/4}, \tag{32}$$

where the a is defined in terms of A,

$$a \equiv \sqrt{\frac{A}{6}}. \tag{33}$$

We note that for a positive c in Eqs. (30)– (32) β's do not give localized graviton solutions near the brane B1, and for a negative c β's have naked singularities at $|y| = -c/4k$ or $-c/4a$. Therefore, to get the effective four dimensional gravity or to avoid the sigularities in the bulk, it is indispensable to cut the extra dimension such that it has a finite length size by introducing another brane, say B2. Thus, we need at least two branes and the situation is similar to that of the RSI except for the 4 form field contributions. Since the extra dimension is finite, the effective four dimensional Planck mass $M_P \equiv M^3 \int dy \beta^2$ is also finite. If we introduce two branes, we should satisfy the boundary conditions at the two branes, consistently with the S^1/Z_2 orbifold symmetry,

$$\frac{d^2}{dy^2}\beta(|y|) = \frac{d^2}{dy^2}\beta(|y|)\Big|_{y\neq 0} + 2\left(\delta(y) - \delta(y-y_c)\right)\frac{d}{d|y|}\beta(|y|). \tag{34}$$

Then the boundary conditions for the above three cases are

$$\text{for} \quad \Lambda_b < 0 \quad : \quad c \; = \; \coth^{-1}\left(\frac{k_1}{k}\right) = 4ky_c - \coth^{-1}\left(\frac{k_2}{k}\right) \tag{35}$$

$$\text{for} \quad \Lambda_b > 0 \quad : \quad c \; = \; \cot^{-1}\left(\frac{k_1}{k}\right) = 4ky_c - \cot^{-1}\left(\frac{k_2}{k}\right) \tag{36}$$

$$\text{for} \quad \Lambda_b = 0 \quad : \quad c \; = \; \frac{a}{k_1} = a\left(4y_c - \frac{1}{k_2}\right), \tag{37}$$

where k_2 is defined in terms of the brane tension Λ_2 at B2,

$$k_2 \equiv \frac{\Lambda_2}{6}. \tag{38}$$

We note that in the case of the H^2 term (not $1/H^2$) in the action, the one-fine-tuning relations between k_1 and k_2 appear always, e.g. the relations (35,36,37), while in the case of RSI model, the two-fine-tuning relations $k = k_1 = -k_2$ were inevitable.

If we introduce both $1/H^2$ and H^2 in the action, there does not exist a flat space self-tuning solution. In this case, the derivative of the warp factor satisfies

$$\beta' = \pm\sqrt{\bar{\Lambda} + \frac{\Lambda_b \beta^2}{6} - \frac{\beta^8}{6A} + \frac{A}{6\beta^8}}. \tag{39}$$

A necessary condition for the existence of a flat space solution is $\beta' \to 0$ and $\beta \to 0$ as $y \to \infty$. Certainly, Eq. (39) does not satisfy this necessary condition. On the other hand, a necessary condition for the de Sitter space regular horizon is that $\beta' =$ finite as $\beta \to 0$. Also, this condition is not met in Eq. (39) and hence there does not exist de Sitter space regular horizon.

C. Comparison with other tuning solutions

There appeared several self-tuning solutions since early eighties [3, 5, 14, 19]. The early stage scenario [3, 5] used field strength of three index antisymmetric tensor field $H_{\mu\nu\rho\sigma}$ to introduce an integration constant. The vacuum value of $\langle H_{\mu\nu\rho\sigma}\rangle = \epsilon_{\mu\nu\rho\sigma}c$ introduces an integration constant c which contributes to the cosmological constant $\propto c^2$. Therefore, there exists a value of c such that the effective cosmological constant vanishes for a range of bare cosmological constant. Once c is determined to give the zero effective cosmological constant, c cannot change since it is a constant. In this sense, the four dimensional example is not a working model. Thus, the self-tuning solution needed a dynamical field to propagate. The three index antisymmetric field is not a dynamical field in 4D spacetime.

Introduction of the extra dimension opened a new game in the self-tuning solutions of the cosmological constants [1, 14, 17, 18, 19, 25]. Here, we briefly comments on the key points of these solutions.

The California self-tuning solution [14, 15] must use the RSI model set up, i.e. introduce two branes B1 and B2, and introduce a specific form for the potential of a bulk scalar field ϕ coupling to the brane matter with a desired form. But the model has a naked singularity and to cure the problem, one has to introduce a brane at this singular point. Then, one needs a condition at the new brane and leads to one fine-tuning there [15]. If one satisfies this one-fine-tuning condition between parameters in the Lagrangian, there always exists a flat space solution. Therefore, the solution is not a self-tuning solution originally anticipated. One interesting or (disastrous to some) point of this model is that it does not allow the de Sitter or anti de Sitter space solutions. In this sense, it is an improvement over the original proposal [3, 5] for the self-tuning solutions. However, it may be difficult to obtain a period of inflation since the de Sitter space exponential expansion is not possible. One should see whether the bulk energy momentum tensor satisfies $\rho = -p = -p_5/2$ to have the exponential expansion of the effective 4D space [24]. However, at present it is not known what matter satisfies this kind of equation of state.

There appeared another interesting self-tuning solution [19, 26], which does not introduce any brane. This model assumes a specific form of the 5D scalar potential multiplied to the matter Lagrangian in 5D. Even though the metric gives a localized gravity, the matter fields propagate in the full 5D space since the potential tends to a constant value as $y \to \infty$. It is not certain how we are forbidden to realize the extra dimension in this model.

Then, there are proposals that the de Sitter space solution is phenomenologically acceptable as far as the curvature at present is sufficiently small [16, 17, 18, 25]. In a sense, it also tries to accomodate the current small vacuum energy [27] with the de Sitter space solutions. [On the other hand, note that the quintessence idea is based on the solution of the cosmological constant problem [28].] The way the proposals make the vacuum energy small is to separate the distance between branes sufficiently large since the vacuum energy is exponentially decreasing with the separating distance. The one fine-tuning at B1 is met but the second fine tuning at B2 is not satisfied and allows the de Sitter space solutions. To have the vacuum energy decreased down to the current energy density, one needs that the separation distance increases as t increases. Namely, the horizon point at the y axis is required to increase. Then it is possible to relate the current vacuum energy with the current mass energy. In Ref. [25], the relation $\Omega_\Lambda = (5/2)\Omega_m$ is obtained for $\Omega_{total} = 1$.

The model presented in [1] allows a self-tuning flat space solution, self-tuning de Sitter and anti de Sitter space solutions. In general it is possible to introduce a period of inflation.

The transition from one flat space to another flat space can arise following the solutions of the Einstein and field equations, as we discuss in the subsequent sections.

III. DE SITTER SPACE SOLUTIONS

We find that there exist de Sitter space solutions in our model with a simple time dependence. First, let us briefly discuss the de Sitter space solution [22] in the RS II model and present de Sitter space solutions in our model.

A. The RS-II model

Let us consider the following metric ansatz for the dS_4 brane in the RS II model:

$$\begin{aligned} ds^2 &= \beta^2(y)[-dt^2 + e^{2\sqrt{\bar{\Lambda}}t}\delta_{ij}dx^i dx^j] + dy^2, \\ &= \beta^2(y)\hat{g}_{\mu\nu}dx^\mu dx^\nu + dy^2 \end{aligned} \tag{40}$$

from which the 4D Ricci tensor is given as $R^{(4)}_{\mu\nu} = 3\bar{\Lambda}\hat{g}_{\mu\nu}$. Then, the warp factor $\beta(y)$ of the dS_4 brane solution is given by

$$\begin{aligned} \beta(y) &= \frac{\sqrt{\bar{\Lambda}}}{k}\sinh[k(y_m - y)], \\ y_m &= \frac{1}{k}\coth^{-1}\left(\frac{k_1}{k}\right) \end{aligned} \tag{41}$$

where the integration constant y_m is determined from the boundary condition at the brane and

$$k \equiv \sqrt{-\frac{\Lambda_b}{6}}, \quad k_1 \equiv \frac{\Lambda_1}{6}. \tag{42}$$

For a positive tension brane, we get a positive value of y_m, which gives rise to an event horizon at the finite proper distance away from the brane. Therefore, the region beyond the horizon can not be causally connected to the region where the brane resides. This bulk horizon resembles the dS_4 horizon on the brane. And, the dS_4 solution is regular along the bulk because the curvature tensors become finite at the bulk horizon. To say about whether the dS_4 solution could describe a dynamical compactification of the extra dimension with the region beyond the horizon cut off, we should check its consistency from the 4D effective cosmological constant by integrating out the fifth dimension. The 4D effective action is given by

$$\begin{aligned} S_{4,eff} &\simeq \int d^4x\sqrt{-g^{(4)}}\int_{-y_m}^{y_m} dy\,\beta^4\Big[\frac{1}{2}R^{(5)} - \Lambda_b - \Lambda_1\delta(y)\Big], \\ &\simeq \int d^4x\sqrt{-g^{(4)}}\Big(\frac{M^2_{P,eff}}{2}R^{(4)} - 6\Lambda_{eff}\Big) \end{aligned} \tag{43}$$

where $g^{(4)}$ and $R^{(4)}$ are given from $g_{\mu\nu} = \hat{g}_{\mu\nu} + h_{\mu\nu}$ in the zero mode expansion and the 4D effective Planck mass and the 4D effective cosmological constant are determined as follows,

$$M^2_{P,eff} = \frac{\bar{\Lambda}}{k^3}\Big(- ky_m + \frac{1}{2}\sinh(2ky_m)\Big) > 0 \tag{44}$$

and

$$\Lambda_{eff} = \frac{1}{3}\int_{-y_m}^{y_m} dy\, \beta^4\Big(4\frac{\beta''}{\beta}+6\Big(\frac{\beta'}{\beta}\Big)^2+\Lambda_b+\Lambda_1\delta(y)\Big),$$
$$= \frac{\bar{\Lambda}^2}{k^3}\Big(-ky_m+\frac{1}{2}\sinh(2ky_m)\Big) > 0. \tag{45}$$

Then, when we compare the 4D Einstein equations of motion derived from the above 4D effective action with those satisfied by the dS_4 brane solution, we can show that the ratio $\Lambda_{eff}/M_{P,eff}^2$ is exactly equal to the 4D cosmological constant $\bar{\Lambda}$ of the brane solution. Therefore, the dS_4 brane solution can be regarded to reproduce the 4D effective de Sitter spacetime with the extra dimension dynamically compactified up to the horizon distance in the bulk.

It is easy to observe that $\Lambda_1 < 0$ does not allow a horizon, which implies a nonlocalized gravity.

B. The KKL model

The background metric is

$$ds^2 = \beta^2(y)\bar{g}_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{46}$$

where the metric corresponds to one of the 4D maximally symmetric $\bar{\Lambda} \neq 0$ spaces:

$$\bar{g}_{\mu\nu} = \mathrm{diag}(-1, e^{2\sqrt{\bar{\Lambda}}t}, e^{2\sqrt{\bar{\Lambda}}t}, e^{2\sqrt{\bar{\Lambda}}t})\ , \quad (dS_4 \text{ background},\ \bar{\Lambda} > 0),$$

for de Sitter space, and

$$\bar{g}_{\mu\nu} = \mathrm{diag}(-e^{2\sqrt{-\bar{\Lambda}}x_3}, e^{2\sqrt{-\bar{\Lambda}}x_3}, e^{2\sqrt{-\bar{\Lambda}}x_3}, 1)\ , \quad (AdS_4 \text{ background}, \bar{\Lambda} < 0),$$

for anti-deSitter space.

The 4D Ricci tensor is given as $\bar{R}_{\mu\nu} = 3\bar{\Lambda}\bar{g}_{\mu\nu}$. And, components of the five dimensional Einstein tensor are

$$G_{\mu\nu} = \Big[3\Big(\frac{\beta''}{\beta}\Big)+3\Big(\frac{\beta'}{\beta}\Big)^2-3\bar{\Lambda}\beta^{-2}\Big]\beta^2\bar{g}_{\mu\nu}, \tag{47}$$
$$G_{55} = 6\Big(\frac{\beta'}{\beta}\Big)^2-6\bar{\Lambda}\beta^{-2}. \tag{48}$$

where $\bar{\Lambda} > 0$ for dS_4 and $\bar{\Lambda} < 0$ for AdS_4 background. The $\bar{\Lambda}$ term arises from the time derivatives of the metric. Then, the (55) and $(\mu\nu)$ components of the Einstein's equations with $1/H^2$ follows,

$$6\Big(\frac{\beta'}{\beta}\Big)^2-6\bar{\Lambda}\beta^{-2} = -\Lambda_b-\frac{\beta^8}{A}, \tag{49}$$
$$3\Big(\frac{\beta'}{\beta}\Big)^2+3\Big(\frac{\beta''}{\beta}\Big)-3\bar{\Lambda}\beta^{-2} = -\Lambda_b-\Lambda_1\delta(y)-3\frac{\beta^8}{A}. \tag{50}$$

We can obtain bulk solutions by solving the (55) component.

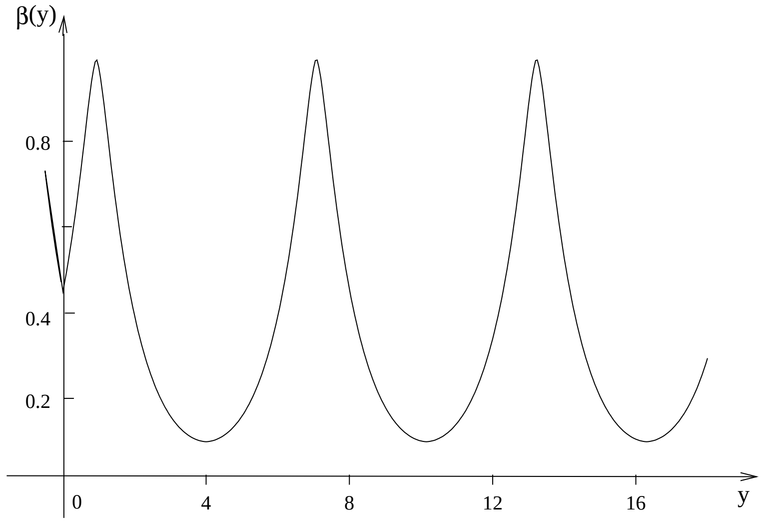

FIG. 2: $\beta(y)$ as a function of y for the anti de Sitter space solution. It is plotted for $\bar{k}^2 = -0.01$, $k = 1$ and $a = 1$, where $\bar{k}^2 = \bar{\Lambda}$.

We can consider two cases: $\beta = 0$ at $y = y_m$=(finite) or $\beta \to 0$ as $y \to \infty$. Otherwise, the 4D Planck mass diverges and localizable gravity does not follow. The case $\beta = 0$ at a finite y arises in the deSitter space solution.

For the anti de Sitter space, let us consider the asymptotic behavior of the warp factor $\beta(y)$ as $y \to \infty$. With $z = \frac{1}{y}$, we can rewrite the (55) component as,

$$\left(\frac{d\beta}{dz}\right)^2 = \frac{\bar{\Lambda}}{z^4} + \frac{k^2\beta^2}{z^4} - \frac{a^2\beta^{10}}{z^4}, \tag{51}$$

where

$$k^2 \equiv -\frac{\Lambda_b}{6}, \;\; a^2 \equiv \frac{1}{6A}. \tag{52}$$

To get the finite 4D Planck mass from the non-compact extra dimension or have the localized gravity, β should be zero as $z \to 0$. However, the 4D cosmological constant term should be divergent as $z \to 0$ even if the last two terms are set to be zero. Therefore, there does not appear the localized gravity for the AdS background with non-zero effective 4D cosmological constants.

In the KKL model [1] the de Sitter space metric ansatz gives β' as

$$\beta' = \sqrt{\bar{k}^2 + k^2\beta^2 - a^2\beta^{10}} \tag{53}$$

where

$$\bar{k} = \sqrt{\frac{\bar{\Lambda}}{6}}, \quad k = \sqrt{\frac{-\Lambda_b}{6}}. \tag{54}$$

The boundary condition at $y = 0$ relates the brane tension (k_1) and the bulk cosmological constant,

$$k_1 = \sqrt{k^2 - a^2\beta^8(0) + \frac{\bar{\Lambda}}{\beta(0)^2}} \tag{55}$$

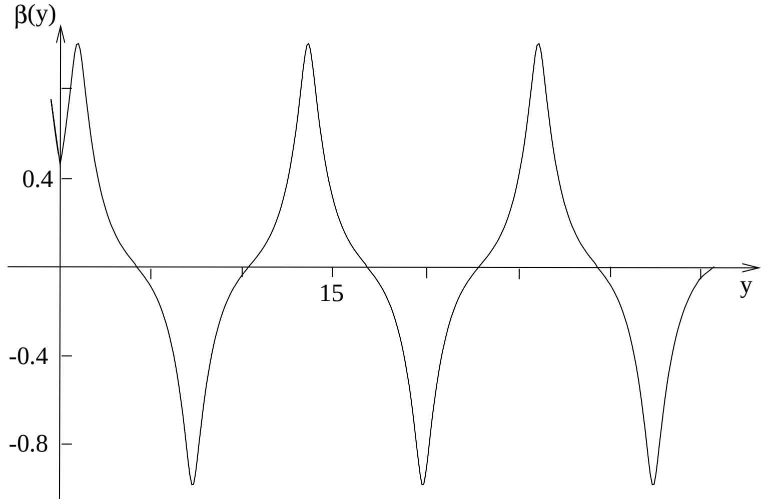

FIG. 3: $\beta(y)$ as a function of y for the de Sitter space solution. It is plotted for $\bar{k}^2 = 0.01$, $k = 1$ and $a = 1$, where $\bar{k}^2 = \bar{\Lambda}$.

where $\beta(0)$ is a function of an integration constant c. The above relation (55) is valid for both the de Sitter and anti de Sitter space solutions.

Consider the half space $y > 0$. If we choose the minus sign in front of the square root of Eq. (53), there always exist y_m as in the RSII model.

Even if we choose the plus sign in front of the square root of Eq. (53) near $y = 0^+$, there exist horizons. This is because there can exist a point where β' vanishes for a sufficiently large β in the region where β increases. For this not to be realized, β should tend to an asymptotic limit as $y \to \infty$. If it goes to an asymptotic limit, the Planck mass becomes infinite and there does not result a localized gravity. Therefore, we restrict to the case for β turning around and coming down, in which case there results a horizon which is determined by the point where $\beta = 0$ at $y = y_m$. Between $y = 0$ and $y = y_m$, there can ($\Lambda_1 < 0$) or cannot ($\Lambda_1 > 0$) exist a point where $\beta' = 0$. Even if there exists such a point as in the above example, the integral $\int d\beta/\sqrt{\bar{k}^2 + k^2\beta^2 - a^2\beta^{10}}$ always converges, and we obtain a localized gravity with the horizon at $y = y_m$. Therefore, there always exist de Sitter space solutions, which have the periodic form as shown in Fig. 3. Similarly, there exist anti de Sitter space solutions, which have the periodic form as shown in Fig. 2. But the anti de Sitter space solution does not lead to a localized gravity. Even if we allow the anti de Sitter space solutions, neglecting the unphysical unlocalized gravity, the probability to choose the anti de Sitter space is exponentially small [5].

The curved space solutions in the KKL model have the effective bulk cosmological constant more negative compared to the RSII case. Therefore, the bound from the weaker energy condition on the background matter supporting those spacetimes is completely satisfied. On the other hand the weaker energy condition is saturated in the RSII case [22]. The weaker energy condition reads $T_{MN}\xi^M\xi^N \geq 0$ for a null vector ξ^M, which corresponds to $(\beta'/\beta)' \leq -\bar{\Lambda}\beta^{-2}$ for the background metric Eq. (46). In the KKL case, the weaker energy condition is always satisfied without the possibility of saturation,

$$\left(\frac{\beta'}{\beta}\right)' = -\bar{\Lambda}\beta^{-2} - 4a^2\beta^8 < -\bar{\Lambda}\beta^{-2}. \tag{56}$$

Existence of de Sitter space solutions in the KKL model allows possible cosmological constants in 4D space as *nonnegative*. Hawking's probabilistic interpretation of the cosmological constant chooses the flat space [5]. However, we need a qualification in this statement. Existence of nonlocalizable de Sitter space solution can give a large probability. The Euclidean action can be written schematically as

$$S_E \propto \int d^4x \left(M_P^2 \frac{R_4}{2} + \cdots \right), \tag{57}$$

after the y integration. The de Sitter space solutions have horizons as shown in Fig. 3. If we integrate over the whole region of the y space M_P diverges. But the Planck mass must be from the integration only up to the horizon connected to $y = 0$ brane which we call the first universe. Then M_P is finite in the first universe. However, if we neglect the effect of the brane, the probability to go the the second universe (the universe between the next two horizons) is of the same order as the probability to go to the first de Sitter space universe, because the horizen points appear periodically. Summing up the probabilities to go to either of these de Sitter universes, we would obtain an infinity. Nevertheless, the second universe does not contain the brane, and we have to exclude this possibility of transition to the universes not containing the brane. Therefore, M_P is considered to be finite, and the probability to stay in the flat universe is maximum [5].

IV. TIME-DEPENDENT SOLUTION OF THE SELF-TUNING MODEL

The existence of the self-tuning solution will be a great leap toward the understanding of the vanishing cosmological constant [3, 5]. As shown in the previous section, the action given in Ref. [1] also allows de Sitter and anti de Sitter space solutions.

Note that if there does not exist any de Sitter space and anti de Sitter space solutions but there exist flat space self-tuning solutions then the cosmological constant may become *automatically* zero for a finite range of parameters in the Lagrangian. In this sense, our self-tuning solution does *not* lead to a vanishing cosmological constant *automatically* but the cosmological constant is *probably zero á la* Hawking [5]. However, the automatic solutions have a difficulty in implementing inflationary period which is needed for our sufficiently homogeneous and isotropic universe [29].

It is anticipated that the spontaneous symmetry breaking at the brane proceeds at the electroweak phase transition and the QCD phase transition. Then the effective potential energy will have a time dependence of the form, $\Lambda_1(t)\delta(y)$. According to the shift of the potential energy at the brane, the solution of the field and Einstein equations will have a time-dependence. Certainly, there will exist time-dependent solutions, and we will be interested in solutions *á la* Hawking [5], changing from one flat space solution with an integration constant c_1 corresponding to the brane cosmological constant Λ_{old} to another integration constant c_2 corresponding to the brane cosmological constant Λ_{new}. This situation corresponds to a time dependent energy momentum tensor. This transition from a flat space solution to another flat space solution is through satisfying the field equations and is different from Witten's sudden choice of a flat space solution [3]. There can be time dependent solutions from a flat space to de Sitter or anti de Sitter spaces, but the probability for these transitions is exponentially small compared to a flat to flat transition [5].

To study the time-dependence the metric is taken as

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\eta_{ij}dx^i dx^j + b^2(t,y)dy^2. \tag{58}$$

The Einstein tensors are,

$$G_{00} = g_{00}\left\{-\frac{3}{n^2}\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}\right)+\frac{3}{b^2}\left(\frac{a''}{a}+\frac{a'}{a}\left(\frac{a'}{a}-\frac{b'}{b}\right)\right)\right\}$$

$$\begin{aligned} G_{ii} = g_{ii}\Big\{ &-\frac{1}{n^2}\left[2\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}-\frac{\dot{a}}{a}\left(2\frac{\dot{n}}{n}-\frac{\dot{a}}{a}\right)-\frac{\dot{b}}{b}\left(\frac{\dot{n}}{n}-2\frac{\dot{a}}{a}\right)\right] \\ &+\frac{1}{b^2}\left[\frac{n''}{n}+2\frac{a''}{a}+\frac{a'}{a}\left(2\frac{n'}{n}+\frac{a'}{a}\right)-\frac{b'}{b}\left(\frac{n'}{n}+2\frac{a'}{a}\right)\right]\Big\} \end{aligned} \quad (59)$$

$$G_{55} = g_{55}\left\{-\frac{3}{n^2}\left(\frac{\ddot{a}}{a}-\frac{\dot{a}}{a}\left(\frac{\dot{n}}{n}-\frac{\dot{a}}{a}\right)\right)+\frac{3}{b^2}\frac{a'}{a}\left(\frac{n'}{n}+\frac{a'}{a}\right)\right\}$$

$$G_{05} = 3\left(\frac{\dot{a}}{a}\frac{n'}{n}+\frac{\dot{b}}{b}\frac{a'}{a}-\frac{\dot{a}'}{a}\right)$$

where dot and prime denote differentiations with respect to t and y, respectively.

With the brane tension Λ_1 at the $y=0$ brane and the bulk cosmological constant Λ_b, the energy momentum tensor is

$$T_{MN} = -g_{MN}\Lambda_b - g_{\mu\nu}\delta^\mu_M\delta^\nu_N\Lambda_1\delta(y) + 4\cdot 4!\left(\frac{4}{H^4}H_{MPQR}H_N{}^{PQR}+\frac{1}{2}g_{MN}\frac{1}{H^2}\right), \quad (60)$$

Considering the homogeneous 3D space, nonzero components of H^{MNPQ} are,

$$\begin{aligned} H^{\mu\nu\rho\sigma} &= \tfrac{1}{\sqrt{-g}}\epsilon^{\mu\nu\rho\sigma 5}\partial_5\sigma \\ H^{5ijk} &= \tfrac{1}{\sqrt{-g}}\epsilon^{5ijk0}\partial_0\sigma. \end{aligned} \quad (61)$$

Then, we have

$$\begin{gathered} H^2 = -4!(f^2-h^2), \quad H_{0NPQ}H_0{}^{NPQ} = -3!g_{00}f^2, \\ H_{iNPQ}H_j{}^{NPQ} = -3!g_{ij}(f^2-h^2), \quad H_{5NPQ}H_5{}^{NPQ} = 3!g_{55}h^2, \\ H_{5NPQ}H_0{}^{NPQ} = 3!bnfh, \quad H_{5NPQ}H_i{}^{NPQ} = 0, \end{gathered} \quad (62)$$

where $f^2 = \sigma'^2/b^2$ and $h^2 = \dot{\sigma}^2/n^2$. The T_{MN} appearing in the Einstein equations $G_{MN} = T_{MN}$ are

$$\begin{aligned} T_{00} &= -g_{00}\left(\Lambda_b+\Lambda_1\delta(y)+\tfrac{6}{f^2-h^2}+\tfrac{4h^2}{(f^2-h^2)^2}\right) \\ T_{ij} &= -g_{ij}\left(\Lambda_b+\Lambda_1\delta(y)+\tfrac{6}{f^2-h^2}\right) \\ T_{55} &= -g_{55}\left(\Lambda_b+\tfrac{2}{f^2-h^2}-\tfrac{4h^2}{(f^2-h^2)^2}\right) \\ T_{05} &= \tfrac{4bnfh}{(f^2-h^2)^2}. \end{aligned} \quad (63)$$

The field equation of the four-form field is

$$\partial_M\left[\frac{\sqrt{-g}H^{MNOP}}{H^4}\right] = \partial_0\left[\frac{\epsilon^{0ijk5}\sigma'}{H^4}\right]+\partial_5\left[\frac{\epsilon^{5ijk0}\dot{\sigma}}{H^4}\right]$$

$$= \frac{\epsilon^{0ijk5}}{(4!)^2}\left\{\partial_0\left[\frac{\sigma'}{((\sigma'/b)^2-(\dot\sigma/n)^2)^2}\right]-\partial_5\left[\frac{\dot\sigma}{((\sigma'/b)^2-(\dot\sigma/n)^2)^2}\right]\right\}=0. \tag{64}$$

Note that σ is given, in the static homogeneous 4D case considered above, by

$$\sigma(|y|)=\frac{ab}{4k^2}\sqrt{2A}\sinh(4kb|y|+c), \tag{65}$$

with $2/f^2=\beta^8/A$.

It is very difficult to find a general solution of the above Einstein equations. However, we are interested in the existence of the interpolating solution between two flat space solutions. We anticipate that a flat space solution, viz. Eq. (13), with $c_i=\tanh^{-1}(\Lambda_i/\sqrt{-6\Lambda_b})$ is given, probably by the initial condition or Hawking's choice of the cosmological constant. During a phase transition when Λ_{old} changes to Λ_{new}, we try to show the existence of an interpolating solution connecting two flat space solutions with c_1 and c_2.

Let us consider the case that the brane tension $\Lambda_1=\Lambda_{old}$ changes to $\Lambda_1=\Lambda_{new}$ instantaneously due to a phase transition by *brane matter* at the brane. Then the boundary condition requires a time dependent b,

$$b(t)=(b_{\rm new}-b_{\rm old})\theta(t-t_0)+b_{\rm old}, \tag{66}$$

which gives

$$\beta(|y|,t)=\left(\frac{k}{a}\right)^{1/4}\left[\cosh\left(4kb(t)(|y|+y_1)\right)\right]^{-1/4}, \quad \text{for} \ \ t\neq t_0, \tag{67}$$

where b_{old}, b_{new} and y_1 are constants. c in Eq. (13) is endowed with a time-dependence as given above. The 't' dependence of β generates non-zero time-derivatives of our metric in the Einstein equation. We suppose that the metric dynamics gives rise to the *bulk matter* fluctuation near the vacuum, $T^{(m)\ M}{}_N\equiv{\rm diag}\,(-\rho,\ p,\ p,\ p,\ p_5)$,

$$\rho = \frac{3}{\beta^2}\frac{\dot\beta}{\beta}\left(\frac{\dot\beta}{\beta}+\frac{\dot b}{b}\right) \tag{68}$$

$$p = -\frac{1}{\beta^2}\left[2\frac{\ddot\beta}{\beta}+\frac{\ddot b}{b}-\left(\frac{\dot\beta}{\beta}\right)^2+\frac{\dot b}{b}\frac{\dot\beta}{\beta}\right] \tag{69}$$

$$p_5 = -\frac{3}{\beta^2}\left(\frac{\ddot\beta}{\beta}\right). \tag{70}$$

Note that the bulk matter contribution vanishes when $t\neq t_0$, because the matter would be proportional to $\delta(t-t_0)$, $\delta^2(t-t_0)$ and $\dot\delta(t-t_0)$.

If the following relation is satisfied,

$$\left(\frac{\beta'}{\beta}\right)^{\cdot}=\frac{\dot b}{b}\frac{\beta'}{\beta}, \tag{71}$$

which leads to $G_{05}=0$, then the 5D continuity equations are *automatically* satisfied [24],

$$\dot\rho+3\frac{\dot\beta}{\beta}(\rho+p)+\frac{\dot b}{b}(\rho+p_5)=0, \tag{72}$$

$$p_5'+3\frac{\beta'}{\beta}(p_5-p)+\frac{\beta'}{\beta}(\rho+p_5)=0, \tag{73}$$

We will see that Eq. (71) is satisfied for our solution.

The 't' dependence of b in Eq. (66) gives also the 't' dependence of the four form field, $H^{MNPQ} \equiv \epsilon^{MNPQR}\partial_R\sigma/\sqrt{-g}$, where σ is

$$\sigma(|y|,t) = \frac{a}{4k^2}\sqrt{2A}\ b(t)\ \sinh\left(4kb(t)(|y|+y_1)\right), \tag{74}$$

where A is the constant. The 't' dependence of σ generates also non-zero H^{ijk5} component of the four form field in Eq. (61), that would be proportional also to the delta function, $\dot\sigma \propto h \propto \delta(t-t_0)$. Since the h term on the RHS of the Einstein equations appear as $h^m/(f^2-h^2)^{n/2}$ with $0 \le m < n$, the $\delta(t)$-function of h gives vanishing contribution at $t=0$. Thus, it turns out that $T_{05}=0$ for any t. For $t \neq t_0$, Eq. (67) satisfies the Einstein equations with the f^2 dependence on the RHS [1]. Near $t=t_0$, T_{MN} are reduced to those of the RS [12] due to the divergence of $\dot\sigma$ or h (which kills the f^2 dependence), and so the solution β becomes the RS solution [12].

We have shown above three solutions: the flat space solution with $c_1 = \tanh^{-1}(\Lambda_{old}/\sqrt{-6\Lambda_b})$ for $t < t_0$, the RS solution [12] with the fine-tuning $k=k_1$, and the flat space solution with $c_2 = \tanh^{-1}(\Lambda_{new}/\sqrt{-6\Lambda_b})$ for $t > t_0$. The fine-tuning solution at $t=0$ means simply that our solution goes through such an intermediate stage. Namely, to satisfy the field equation and the Einstein equations, Λ_{old} suddenly jumps to $\sqrt{-6\Lambda_b}$ and again suddenly jumps to Λ_{new}. If the transition from Λ_{old} to Λ_{new} is not abrupt, there would exist a solution smoothly connecting the two flat solutions with c_1 and c_2. But the above example shows that at the intermediate stage it would go through the RS solution [12].

The consistency of the solution is achieved by showing Eq. (71) using the divergence of h. As the time derivatives of the metric in $G_{55} = T_{55} + T_{55}^{(m)}$ have been identified already with p_5 of $T_{55}^{(m)}$, the remnant reads as

$$\frac{\beta'}{\beta} = \pm b(t)\sqrt{-\frac{\Lambda_b}{6} - \frac{1}{3}\frac{1}{f^2-h^2} + \frac{2}{3}\frac{h^2}{(f^2-h^2)^2}}, \tag{75}$$

from which we can see immediately that Eq. (71) is satisfied,

$$\begin{aligned} \left(\frac{\beta'}{\beta}\right)^{\cdot} &= \pm\frac{\dot b}{b} b\sqrt{-\frac{\Lambda_b}{6} - \frac{1}{3}\frac{1}{f^2-h^2} + \frac{2}{3}\frac{h^2}{(f^2-h^2)^2}} \\ &\quad \pm b\frac{\partial}{\partial t}\sqrt{-\frac{\Lambda_b}{6} - \frac{1}{3}\frac{1}{f^2-h^2} + \frac{2}{3}\frac{h^2}{(f^2-h^2)^2}} \\ &= \frac{\dot b}{b}\left(\frac{\beta'}{\beta}\right), \end{aligned} \tag{76}$$

where we used

$$\frac{\partial}{\partial t}\left[\frac{1}{\delta^2(t-t_0)}\right] = 0. \tag{77}$$

Note that Eq. (77) can be shown for a specific representation of the delta function.

We should check whether the equation of motion for the four form field Eq. (64) remains satisfied even when the constant b is changed to $b(t)$ like Eq. (66). But we can also see easily that even in the case, it is satisfied always regardless of $t=t_0$ or not. When $t \neq t_0$, it is

the static case, and so the equation of motion is trivially satisfied. On the other hand, for $t \approx t_0$ the equation of motion is also satisfied due to the $\delta(t - t_0)$ divergence of h, which is checked easily if we use Eq. (77).

We have seen that there exists a time-dependent solution when the brane tension changes from Λ_{old} to Λ_{new} instantaneously. In this case, the universe starting from a flat universe can go to a flat universe instantaneously. Instead of the sudden shift of the brane tension, the transition from Λ_{old} to Λ_{new} can be smooth. We argue that in this case also, there exists a solution since we can replace the δ function with a smooth function $D(t;\epsilon)$ of t defined in a short interval ϵ, such that $\lim_{\epsilon\to 0} D(t;\epsilon) = \delta(t)$. The error by introducing ϵ is at most ϵ and hence our solution is approximate, but this shows that there can exists a smooth solution very close to our approximate solution with an error of order ϵ.

There may exist solutions connecting de Sitter space and flat space, de Sitter space and de Sitter space, etc. [30]. But the final universe may be chosen probabilistically to be the flat universe *á la* Hawking [5].

V. CONCLUSION

We considered the cosmological constant in the $(4+1)$-dimension with a warp factor. A brane, containing the matter fields, is located at the origin $y = 0$ of the extra dimension. To separate the 4D space we introduced a three index antisymmetric tensor field with the field strength H. The simple form for the H action is considered: the Lagrangian with $1/H^2$ and H^2. We found a self-tuning solution, Eq. (13), with $1/H^2$. The H^2 term does not allow a self-tuning solution, but allows nonlocalizable gravity with one brane or one-fine-tuning solutions with two branes.

We concentrated the discussion related to the self-tuning solution, Eq. (13). We also found that there exist de Sitter space and anti de Sitter space solutions with $1/H^2$ term. For a finite range of parameters of the bulk cosmological constant Λ_b and the brane tension Λ_1, there always exists the flat space solution but it is not unique. Therefore, when the boundary condition is changed at the electroweak or QCD phase transitions, the cosmological constant after the phase transitions can be anything allowed by the Einstein equations. However, the probability to choose the vanishing cosmological constant is infinitely large compared to the others' [5]. This argument is applicable in our case since there exists the self-tuning solution, Eq. (13).

This may not sound so attractive as any model not allowing de Sitter or anti de Sitter space solutions. However, if there exists a such model, then it may not allow 'inflation' which is probably needed in cosmology. In the self-tuning model the transition from one integration constant to another integration constant is possible through satisfying the equations of motion.

For the self-tuning solution we presented to work in cosmology, there must be a natural explanation of the current acceleration of the expansion [27]. There appeared some proposals [28], but these must be shown to work with the self-tuning solution.

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[1] J. E. Kim, B. Kyae and H. M. Lee, hep-th/0011118 (2000).

[2] M. Veltman, Phys. Rev. Lett. **34** (1975) 777; For a review, see, S. Weinberg, Rev. Mod. Phys. **61** (1989) 1.

[3] E. Witten, *Proc. Shelter Island II Proceedings* (Shelter Island, 1983), ed. R. Jackiw, H. Khuri, S. Weinberg and E. Witten (MIT Press, 1985) p369.

[4] E. Baum, Phys. Lett. **B138** (1983) 185.

[5] S. Hawking, Phys. Lett. **B134** (1984) 403.

[6] S. Coleman, Nucl. Phys. **B310** (1988) 643.

[7] A. Linde, Phys. Lett. **B200** (1988) 272.

[8] M. Duff, Phys. Lett. **B226** (1989) 36.

[9] S. Weinberg, Phys. Rev. Lett. **59** (1987) 2607; S. M. Barr, J. F. Donoghue and D. Seckel, Phys. Rev. **D57** (1998) 5480; T. Banks, M. Dine and L. Motl, hep-th/0007206.

[10] G. Dee and J. S. Langer, Phys. Rev. Lett. **50** (1983) 383; R. C. Brower, D. A. Kessler, J. Koplik and H. Levine, Phys. Rev. Lett. **51** (1983) 1111.

[11] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370 [hep-th/9905221].

[12] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 4690 [hep-th/9906064].

[13] V. A. Rubakov and M. E. Shaposhinikov, Phys. Lett. **B125** (1983) 139.

[14] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, and R. Sundrum, Phys. Lett. **B480** (2000) 193 [hep-th/0001197]; S. Kachru, M. Schulz and E. Silverstein, Phys. Rev. **D62** (2000) 045021 [hep-th/0001206]; C. Csaki, J. Erlich, C. Grojean and T. Hollowood, Nucl. Phys. **B584** (2000) 359 [hep-th/0004133].

[15] S. Forste, Z. Lalak, S. Lavignac and H. P. Nilles, Phys. Lett. **B481** (2000) 360 [hep-th/0002164] and JHEP **0009** (2000) 034 [hep-th/0006139].

[16] H. B. Kim, Phys. Lett. **B478** (2000) 285.

[17] S.-H. H. Tye and I. Wasserman, hep-th/0006068; E. Flanagan, N. Jones, H. Stoica, S.-H. H. Tye, and I. Wasserman, hep-th/0012129.

[18] J. M. Cline and H. Firouzjahi, hep-ph/0012090; N. Tetradis, hep-th/0012106.

[19] A. Kehagias and K. Tamvakis, hep-th/0011006.

[20] J. E. Kim, B. Kyae and H. M. Lee, Phys. Rev. **D62** (2000) 045013 [hep-ph/9912344] and Nucl. Phys. **B582** (2000) 296 [hep-th/0004005]; I. Low and A. Zee, Nucl. Phys. **B585** (2000) 395 [hep-th/0004124].

[21] P. G. O. Freund and M. A. Rubin, Phys. Lett. **B97** (1980) 233.

[22] A. Karch and L. Randall, hep-th/0011156. See, earlier papers: T. Nihei, Phys. Lett. **B465** (1999) 81 [hep-ph/9905487]; H. B. Kim and H. D. Kim, Phys. Rev. **D61** (2000) 064003 [hep-th/9909053].

[23] T. Padmanabhan and S. Shankaranarayanan, hep-th/0011159.

[24] J. E. Kim and B. Kyae, Phys. Lett. **B486** (2000) 165 hep-th/0005139.

[25] Ph. Brax and A. Davis, hep-th/0011045.

[26] Z. Kakushadze, Phys. Lett. **B489** (2000) 207, hep-th/0006215.

[27] The Supernova Cosmology Project, Bull. Am. Astron. Soc. **5** (1998) 1351.

[28] P. Binetruy, Phys. Rev. **D60** (1999) 063502; C. Kolda and D. H. Lyth, Phys. Lett. **B458**

(1999) 197; J. E. Kim, JHEP **9905** (1999) 022 and **0006** (2000) 016; T. Chiba, Phys. Rev. **D60** (1999) 083508; P. Brax and J. Martin, Phys. Lett. **B468** (1999) 40; A. Masiero, M. Pietroni and F. Rosati, Phys. Rev. **D61** (2000) 023504.

[29] A. Guth, Phys. Rev. **D23** (1981) 347.

[30] G. T. Horowitz, I. Low and A. Zee, Phys. Rev. **D62** (2000) 086005 [hep-th/0004206]; P. Binetruy, J. M. Cline and C. Grojean, Phys. Lett. **B489** (2000) 403 [hep-th/0007029].